臺灣綜合大學系統111學年度學士班轉學生聯合招生考試試題

科目名稱	工程數學	類組代碼	D37
		科目碼	D3792

※本項考試依簡章規定所有考科均「不可」使用計算機。　　本科試題共計　1頁

1. (15%)

   (a) Please make the comparison between Green's theorem ($\oint_C \mathbf{F}\cdot d\mathbf{r} = \iint_R (\text{curl }\mathbf{F})\cdot\mathbf{k}\, dA$) and Stokes' theorem ($\oint_C \mathbf{F}\cdot d\mathbf{r} = \iint_S (\text{curl }\mathbf{F})\cdot\mathbf{n}\, dS$).

   (b) Verify Stokes' theorem $\mathbf{F}=x\mathbf{i}+y\mathbf{j}+k\mathbf{z}$, and for S being the portion of the sphere $x^2+y^2+z^2=1$ for $z\geq 0$. Assume S is oriented upward.

2. (15%) Find the eigenvalues and eigenvectors of the given matrix. Also, state whether the matrix is singular or nonsingular

$$\mathbf{A}=\begin{bmatrix}1 & 2 & 1\\ 2 & 1 & 1\\ 1 & 3 & 1\end{bmatrix}$$

3. (20%) Evaluate the integrals.

   (a) $\int_0^4\int_0^{\sqrt{y}} xe^{y^2}\,dxdy$

   (b) $\int_0^4\int_0^1\int_0^{x^2}\frac{1}{\sqrt{x^2-y^2}}\,dydxdz$

4. (15%) Find the rank of the given matrix $\begin{cases}x_1-2x_2+x_3=2\\ 3x_1-x_2+2x_3=5\\ 2x_1+x_2+x_3=1\end{cases}$ and determinate how many solutions this system has.

5. (15%) Please solve $\begin{cases}x_1-x_2-2x_3=0\\ 6x_1+3x_3=0\\ 2x_1+4x_2+5x_3=0\end{cases}$ or show that no solution exists.

6. (20%) Please solve the initial value problem (IVP).

   $y'''-2y''-y'+2y=0,\ y(0)=3,\ y'(0)=-1,\ y''(0)=3$