

臺灣綜合大學系統 111 學年度學士班轉學生聯合招生考試試題

科目名稱	微積分 B	類組代碼	共同考科
		科目碼	E0012

※本項考試依簡章規定所有考科均「不可」使用計算機。 本科試題共計 1 頁

* SHOW ALL YOUR WORK and Highlight Your Final Answer *

1. Evaluate the following limits.

(a) (5%) $\lim_{x \rightarrow 0} \frac{4^x - 1}{x}$.

(b) (5%) $\lim_{x \rightarrow -6} \frac{\sqrt{10-x} - 4}{x+6}$.

2. (10%) Compute the integral $\int_0^3 |x^2 - 3x + 2| dx$.

3. (10%) Compute the integral $\int_0^\pi e^{-x} \sin(\pi - x) dx$.

4. (10%) Compute the integral $\int_1^9 \frac{1}{\sqrt{x}(1+\sqrt{x})^2} dx$.

5. (10%) Compute the integral $\int_0^2 \frac{1}{6x^2+7x+2} dx$.

6. (10%) Find the slope of the tangent line to the graph of $y^2(x^2 + y^2) = x^2$ at the point $(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2})$.

7. Define $u(x, y) = y + xe^{xy}$, where $x = 2s + t$ and $y = 2t + 1$.

(a) (5%) Find $\frac{\partial u}{\partial x}$ when $(x, y) = (0, 1)$.

(b) (5%) Find $\frac{\partial u}{\partial t}$ when $(s, t) = (0, 0)$.

8.

(a) (5%) Let $\{a_n\}_{n \in \mathbb{N}}$ be a sequence defined by $a_n = \frac{2^n}{n!}$. Find $\sum_{n=1}^{\infty} na_n$.

(b) (5%) Use the Integral Test to determine whether the series $\sum_{n=1}^{\infty} \frac{1}{n(\ln n)}$ is convergent or divergent?

9. (10%) Find the absolute maxima and absolute minima of

$$f(x, y) = 3x^2 + 2y^2 - 4y$$

on the region R in the xy -plane bounded by the graphs of $y = x^2$ and $y = 4$.

10. (10%) Use Lagrange multipliers to find the extreme values of

$$f(x, y) = \frac{5}{4}x^2 + \frac{7}{4}y^2 - \frac{\sqrt{3}}{2}xy$$

subject to the constraint $x^2 + y^2 = 1$.